DFG-P-PHY

PHYSICS
Paper – I

Time Allowed : **Three** Hours

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

*Answers must be written in **ENGLISH** only.*

Useful Constants :

Electron charge *(e)*	$= 1{\cdot}602 \times 10^{-19}$ C
Electron rest mass (m_e)	$= 9{\cdot}109 \times 10^{-31}$ kg
Proton mass (m_p)	$= 1{\cdot}672 \times 10^{-27}$ kg
Vacuum permittivity (ε_0)	$= 8{\cdot}854 \times 10^{-12}$ farad/m
Vacuum permeability (μ_0)	$= 1{\cdot}257 \times 10^{-6}$ henry/m
Velocity of light in free space *(c)*	$= 3 \times 10^{8}$ m/s
Boltzmann constant *(k)*	$= 1{\cdot}380 \times 10^{-23}$ J/K
Electron volt (eV)	$= 1{\cdot}602 \times 10^{-19}$ J
Planck's constant *(h)*	$= 6{\cdot}626 \times 10^{-34}$ Js
Stefan's constant (σ)	$= 5{\cdot}67 \times 10^{-8}$ Wm^{-2} K^{-4}
Avogadro number *(N)*	$= 6{\cdot}022 \times 10^{26}$ kmol^{-1}
Gas constant *(R)*	$= 8{\cdot}31 \times 10^{3}$ J kmol^{-1} K^{-1}
exp (1)	$= 2{\cdot}718$

SECTION A

Q1. Answer all the following questions : *8×5=40*

(a) A mouse of mass m jumps on the outside edge of a freely turning ceiling fan of rotational inertia I and radius R. By what ratio does the angular velocity change ? *8*

(b) In an He – Ne laser system, the two energy levels of Ne involved in lasing action have energy values of 20·66 eV and 18·70 eV respectively. Population inversion occurs between these two energy levels. What will be the wavelength of the laser beam produced ? What will be the population in the metastable energy level with respect to the upper excited level at room temperature (27°C) ? *8*

(c) How can the refractive index of a liquid be determined using a method of Newton's rings ? Obtain an appropriate expression for calculating the refractive index. *8*

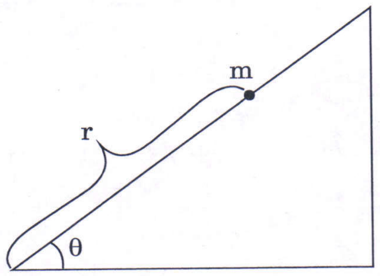
(d) How fast and in what direction must galaxy A be moving if an absorption line found at wavelength 550 nm (green) for a stationary galaxy is shifted to 450 nm for galaxy A and how fast and in what direction is galaxy B moving if the same line is shifted to 700 nm for it ? *8*

(e) The quality factor Q in a damped harmonic motion is defined as

$$Q = \frac{2\pi \times \text{Average energy stored per cycle}}{\text{Average energy dissipated per cycle}}.$$

Show that $Q = \frac{\omega}{2b}$ (where the symbols have usual meaning). *8*

Q2. (a) A particle of mass m rests on a smooth plane. The plane is then raised to an inclination angle θ at a constant rate α (θ = 0 at t = 0), causing the particle to move down the plane, as shown in the figure below.



Write down the Lagrangian and determine the equations of motion. Solve the resulting equation for "r" to obtain the expression for r(t). *20*

(b) A particle moves in an elliptical orbit in an inverse-square-law central force field. If the ratio of the maximum angular velocity to the minimum angular velocity of the particle in the orbit is n, then show that the eccentricity of the orbit is $\varepsilon = \dfrac{\sqrt{n}-1}{\sqrt{n}+1}$. *10*

(c) A three-particle system consists of masses m_i, i = 1, 2, 3 and their respective coordinates (x_i, y_i, z_i) :

$$m_1 = 3m, (x_1, y_1, z_1) = (a, 0, a),$$

$$m_2 = 4m, (x_2, y_2, z_2) = (a, a, -a),$$

$$m_3 = 2m, (x_3, y_3, z_3) = (-a, a, 0),$$

where m and a are positive constants.

Calculate the inertia tensor of the system. *10*

Q3. (a) Consider a light beam passing through a horizontal column of water moving with a velocity 'v'. Determine the speed u of the light measured in the lab frame when the beam travels in the same direction as the flow of the water. (Speed of water = v (lab frame), and refractive index = n) *15*

(b) A High Frequency (HF) radio receiver receives simultaneously two signals from a transmitter 400 km away, one by a path along the surface of the Earth, and the other by reflection from a portion of the ionospheric layer situated at a height of 200 km. We assume that the Earth is flat and the ionospheric layer acts as a perfect horizontal reflector, which is moving slowly in the vertical direction. When the frequency of the transmitted wave is 10 MHz, it is observed that the combined signal strength varies from maximum to minimum and back to maximum 6 times per minute. With what slow vertical speed is the ionospheric layer moving ? *15*

(c) It is required that a real image twice the size of the object be formed by a thin plano-convex lens. If the lens has a radius of curvature of 50 cm and a refractive index of $\mu = 1{\cdot}5$, determine the locations of the object and image with respect to the lens. *10*

Q4. (a) Explain Huygens' theory of double refraction in a uniaxial crystal. Discuss the formation of ordinary and extraordinary rays for oblique incidence when an optic axis is in the plane of incidence and when it is perpendicular to the plane of incidence. *20*

(b) Two Fabry – Pérot interferometers have same plate separation but the coefficients of intensity reflection are 0·7 and 0·9. Deduce the relative width of the maxima in the above two cases. *10*

(c) With a neat ray diagram, describe Fraunhofer diffraction at a circular aperture. *10*

SECTION B

Q5. Answer all the following questions : *8×5=40*

(a) A charge of $-3{\cdot}30\ \mu C$ is fixed in place. From a horizontal distance of $0{\cdot}0455$ m, a particle of mass $7{\cdot}35 \times 10^{-3}$ kg and charge $-7{\cdot}45\ \mu C$ is fired with an initial speed of $62{\cdot}5$ m/s directly towards the fixed charge. How far does the particle travel before it comes to rest ? *8*

(b) A uniformly charged sphere of radius R carries a total charge Q. Find the net force that the southern hemisphere exerts on the northern hemisphere. Express your answer in terms of the radius R and total charge Q. *8*

(c) Obtain Clausius – Clapeyron equation which applies to any first-order change of phase or any transition that occurs at constant temperature and pressure. Use Maxwell's thermodynamic relation for deriving the equation. *8*

(d) Write down the formula for Planck's radiation law. Show that it reduces to Wien displacement law for shorter wavelengths and to Rayleigh – Jeans law for longer wavelengths. *8*

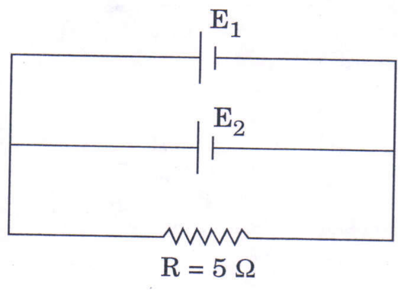
(e) If we consider the Earth as a black body in thermal equilibrium, estimate the global temperature of our planet in terms of the temperature of the Sun, T_{sun}, its radius, R_{sun}, and distance D between the Earth and the Sun. *8*

Q6. (a) (i) Using the laws of transformation of the electric field, $\vec{E}$, and the magnetic field, $\vec{B}$, show that $(E^2 - C^2B^2)$ is relativistically invariant.

(ii) Suppose that in one inertial frame $\vec{B} = 0$ but $\vec{E} \neq 0$ (at some point P). Is it possible to find another inertial frame in which the electric field is zero at P ? *6+4=10*

(b) Consider electromagnetic fields in a medium with permittivity ε and permeability μ. The fields are space and time dependent. Write down the full system of Maxwell's equations along with the constitutive equations. Then, derive the continuity equation from them. Using this continuity equation, show that the total charge of the universe is conserved. *15*

(c) Two cells E_1 of E.M.F. 2 volts and internal resistance 1·5 ohms and E_2 of E.M.F. 1·5 volts and internal resistance 1 ohm, are joined in parallel with like poles together (see the figure below). Calculate the current that would pass through a 5-ohm resistance joined in parallel with the cells. *15*



Q7. (a) Derive an expression for the entropy change in the expansion of a gas from volume V_i to volume V_f. Use a statistical definition of entropy for derivation. *20*

(b) The equation of state of a dilute gas at very high temperature is described by

$$\frac{PV}{kT} = 1 + \frac{B(T)}{V},$$

where V is the volume per particle and B(T) is a negative quantity. One can conclude that this is a property of a Van der Waal's gas. Explain why it is a property of Van der Waal's gas. *10*

(c) State and prove Stefan – Boltzmann law for ideal gases in thermodynamics. *10*

Q8. (a) Obtain an expression for the specific heat capacity of a solid on the basis of Einstein's theory. How far do the results from this theory agree with experimental data ? *15*

(b) Derive Maxwell's thermodynamic relations using concepts of internal energy, Helmholtz function, Gibbs' function and enthalpy. *10*

(c) Consider a quantum – mechanical gas of non-interacting spin zero bosons, each of mass m which are free to move within volume V.

(i) Find the energy and heat capacity in the very low temperature region. Discuss why it is appropriate at low temperatures to put chemical potential equal to zero.

(ii) Show how the calculation is modified for a photon (mass = 0) gas. Prove that the energy is proportional to T^4. *5+10=15*