STATISTICS

Paper – I

Time Allowed : **Three** Hours	Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

*Answers must be written in **ENGLISH** only.*

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

SECTION A

Q1. (a) (i) For n events $A_1, A_2, \ldots, A_n$, show that

$$P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1).$$

(ii) Let $\{A_n\}$ be an increasing sequence of sets (events), then show that

$$\lim_{n \to \infty} P(A_n) = P\left(\lim_{n \to \infty} A_n\right) = P\left(\bigcup_{n=1}^{\infty} A_n\right).$$

3+5

(b) Let $\Omega = \{1, 2, 3, 4\}$ and $p_i = P\{i\}$, $i = 1, 2, 3, 4$.

Assume that

$$p_1 = \frac{\sqrt{2}}{2} - \frac{1}{4},$$

$$p_2 = \frac{1}{4},$$

$$p_3 = \frac{3}{4} - \frac{\sqrt{2}}{2}, \text{ and}$$

$$p_4 = \frac{1}{4}.$$

Define the events

$$E_1 = \{1, 3\},\ E_2 = \{2, 3\} \text{ and } E_3 = \{3, 4\}.$$

Check whether E_1, E_2 and E_3 are mutually independent. *8*

(c) Suppose that $X_1, \ldots, X_n$ form a random sample from a Uniform distribution on the interval $[\theta_1, \theta_2]$ where both θ_1 and θ_2 are unknown $(-\infty < \theta_1 < \theta_2 < \infty)$. Find the maximum likelihood estimators of θ_1 and θ_2. *8*

(d) Suppose that $X_1, \ldots, X_n$ form a random sample from a Gamma distribution for which the value of parameter α is unknown $(\alpha > 0)$ and value of parameter β is known. Show that the joint probability density function of $X_1, \ldots, X_n$ has a Monotone Likelihood Ratio (MLR). *8*

(e) (i) Let $\{X_n\}$ be a sequence of events such that

$$P[X_n = n^2 - 1] = \frac{1}{n^2},$$

$$P[X_n = -1] = 1 - \frac{1}{n^2}.$$

Check whether Strong Law of Large Numbers holds. Also comment about Weak Law of Large Numbers.

(ii) Let

$X_n \xrightarrow{\text{prob.}} X$ and $Y_n \xrightarrow{\text{prob.}} Y$, then show that

$X_n Y_n \longrightarrow XY$ in probability. *8*

Q2. (a) Suppose that a diagnostic test for HIV(+) status has both sensitivity (P(Test positive | Disease)) and specificity (P(Test negative | No disease)) equal to 0·95 and the real possibility (P(Disease)) is 0·005. Find the probability that a subject is truely HIV(+) given that the diagnostic test is positive. *8*

(b) Let a continuous random variable X have probability density function (pdf) given by

$$f(x) \begin{cases} \dfrac{2x}{\pi^2}, & 0 < x < \pi \\ 0, & \text{otherwise} \end{cases}$$

Find the probability density function of $Y = \sin X$. *10*

(c) Let (X, Y) be uniformly distributed over

$$R = \{(x, y) \mid x^2 + y^2 \le 1,\ y \ge 0\}$$

Find

(i) the distributions of $(X \mid Y = y)$ and $(Y \mid X = x)$,

(ii) $E(Y \mid X = x)$ and $E(X \mid Y = y)$, and

(iii) Correlation coefficient ρ_{XY}. *12*

(d) Let X follow Pareto distribution with parameters α and θ with probability density function (pdf)

$$f(x) = \frac{\alpha\, \theta^{\alpha}}{(x + \theta)^{\alpha + 1}}, \quad x > 0,$$

then show that

$$Y = ln\left(\frac{X}{\theta}\right)$$

follows Logistic distribution. *10*

Q3. (a) Suppose that $x_1, ..., x_n$ form a random sample from a Beta distribution with parameters α and β where the value of α is known and the value of β is unknown ($\beta > 0$). Obtain a sufficient statistic for β. *8*

(b) Let $x_1, ..., x_n$ be a random sample from $P(\theta)$. Find Uniformly Minimum Variance Unbiased Estimator (UMVUE) of $e^{-\theta}$. Compute the estimator based on the following sample observations : *12*

1, 3, 0, 8, 5, 6, 9, 2, 7, 5

(c) The lifetimes of fluorescent lamps are independent exponential random variables with parameter β. Suppose that β has a prior distribution Gamma with parameters 4 and 20,000. After we observe 5 lamps with lifetimes 2911, 3403, 3237, 3509 and 3118 (in hours), we want to predict the lifetime X_6 of the next lamp. Obtain the predictive distribution of X_6. *10*

(d) Suppose $X_1, X_2, ..., X_n \sim N(\mu, \sigma^2)$ where σ^2 is known.

(i) Find the Likelihood Ratio Test (LRT) for $H_0 : \mu \leq \mu_0$ vs $H_1 : \mu > \mu_0$.

(ii) Show that the test in (i) is a UMP test. *10*

Q4. (a) For any sequence $\{A_n, n \geq 1\}$ of events with $\sum_{k=1}^{\infty} P(A_k) < \infty$, comment on the following with justification : *4+6*

(i) $P\left(\bigcup_{k=n}^{\infty} A_k\right) \leq \sum_{k=n}^{\infty} P(A_k)$

(ii) $P(\limsup_{n} A_n) = 0$

(b) Suppose that an examination contains 99 questions arranged in a sequence from the easiest to the most difficult. Suppose that the probability that a particular student will answer the first question correctly is 0·99, the probability that he will answer the second question correctly is 0·98 and in general, the probability that he will answer the i^{th} question correctly is $1 - \frac{i}{100}$, for $i = 1, 2, ..., 99$. It is assumed that all questions will be answered independently and that the student must answer at least 60 questions correctly to pass the examination. What is the probability that the student will pass ? *10*

(Tables 1(a) and 1(b) are provided at the end.)

(c) Let $\{X_0, X_1, ...\}$ be a Markov Chain (MC) with transition probability matrix

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & 1-p & 0 \end{bmatrix} \end{array}, \quad 0 < p < 1$$

Let $g(x) = \begin{cases} 0, & x = 1 \\ 1, & x = 2, 3. \end{cases}$

If $Y_n = g(X_n)$, $n \geq 0$, show that $\{Y_0, Y_1, ...\}$ is not a Markov Chain (MC). *10*

(d) Derive Kolmogorov – Smirnov test for two samples and illustrate. *10*

SECTION B

Q5. (a) Let N be the incidence matrix of a Balanced Incomplete Block Design (BIBD) of order $b \times v$. Show that $b \geq v$. *8*

(b) (i) A plane is fitted to $n = 33$ observations on (X_1, X_2, Y) and it is found that overall regression is just significant at $\alpha = 0{\cdot}05$ level. Find out R^2 based on the available information.

(Tables 2(a) and 2(b) are provided at the end.)

(ii) In a one-way layout, show that for all values of i, i′ and j,

$j = 1, 2, \ldots, n,$

$i, i' = 1, 2, \ldots, p,$

$w_1 = y_{ij} - \bar{y}_{i\cdot},$

$w_2 = \bar{y}_{i'\cdot} - \bar{y}_{i\cdot\cdot},$

$w_3 = \bar{y}_{i\cdot}.$

are uncorrelated with each other (under usual assumptions). *3+5*

(c) Let **Y** follow $\mathbf{N}_2(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Find distribution of $\mathbf{Z} = C\mathbf{Y}$ where C is a 2×2 non-singular matrix. Also give an explicit form of matrix C such that $C\Sigma C' = I$, where Σ is the dispersion matrix of **Y**. *8*

(d) Construct a 2^3 design in two blocks where ABC is confounded. *8*

(e) Find the condition under which systematic sample mean is more efficient than a simple random sample mean. *8*

Q6. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from $\mathbf{N}_p(\mu, \Sigma)$. Consider the hypothesis $H_0 : \mu = k\mu_0$, where Σ and μ_0 are known. Derive the MLE for k. Show that -2 log likelihood ratio is

$$n\overline{X}\, \Sigma^{-1}(\Sigma - (\mu_0^1\, \Sigma^{-1}\, \mu_0)^{-1}\, \mu_0\, \mu_0^1)\, \Sigma^{-1}\overline{X}.$$

Deduce the distribution of the statistic. *10*

(b) Suppose that a chemical engineer considers the time of reaction for a chemical process as a function of the type of catalyst used. Four catalysts are being investigated and the procedure consists of selecting a batch of raw materials. The observations recorded are as shown below :

Catalyst	*Batch of Raw Materials*			
	1	2	3	4
1	73	75	68	–
2	75	–	72	75
3	73	74	–	71
4	–	75	67	72

Identify the design and analyse it. *10*

(Tables 2(a) and 2(b) are provided at the end.)

(c) Let X_1 and X_2 be independent random vectors of order $(n_1 \times p)$ and $(n_2 \times p)$ respectively and let n_i rows of X_i $(i = 1, 2)$ be independently and identically distributed as $N_p(\mu_i, \Sigma_i)$.

Show that for $\mu_1 = \mu_2$ and $\Sigma_1 = \Sigma_2$,

$$\frac{n_1 n_2}{n} D^2 \simeq T^2 (p, n - 2),$$

where $n = n_1 + n_2$, D^2 denotes sample Mahalanobis distance statistic and T^2 denotes the Hotelling's T^2. *10*

(d) Find an unbiased estimator of the population mean under probability proportional to size (PPS) sampling with replacement. Find the variance of this estimator and also give an estimator of this variance. *10*

Q7. (a) In a simple linear regression problem $Y = \beta_0 + \beta_1 X + \varepsilon$, $\varepsilon \sim N(0, \sigma^2)$, in which a patient's response Y to a new drug B is to be related to his response X to a standard drug A. Suppose 10 pairs of observations (y_i, x_i), $i = 1 \ldots 10$ are obtained.

(i) Determine the MLEs of $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\sigma}^2$ of their corresponding parameters,

(ii) Obtain variance $(\hat{\beta}_0)$, variance $(\hat{\beta}_1)$ and corr $(\hat{\beta}_0, \hat{\beta}_1)$. *10*

(b) In a two-way layout with k observations in each cell ($k \geq 2$), construct a test of the null hypothesis that all the interactions are zero. (Model and all assumptions are required to be specified in detail). *8*

(c) For the covariance matrix given by

$$\sum = \begin{pmatrix} 1 & 4 \\ 4 & 100 \end{pmatrix},$$

obtain the proportion of the total population variance explained by the first principal component. *10*

(d) In simple random sampling where n paired observations (y_i, x_i), $i = 1 \ldots n$ are drawn, obtain the regression estimators and derive its large sample variance. *12*

Q8. (a) Use the method of Lagrange's multipliers to show that for a least squares problem,

$T = (\mathbf{Y} - X\beta)' (\mathbf{Y} - X\beta) + \lambda'(\mathbf{d} - C\beta)$

is minimized with respect to β and λ where

$\hat{\beta} = \mathbf{b} + (X' X)^{-1} C' [C(X'X)^{-1}C']^{-1} (\mathbf{d} - C\mathbf{b})$

where $\mathbf{b}$ is unrestricted least squares estimator of β. *10*

(b) Let there be two populations Π_1 and Π_2. It is known that about 30% of all objects belong to Π_2 and

$C(2 \mid 1)$: cost incurred when a Π_1 observation is incorrectly classified as Π_2 observation = 15;

$C(1 \mid 2)$: cost incurred when a Π_2 observation is incorrectly classified as Π_1 observation = 10.

Suppose the two density functions $f_1(x)$ and $f_2(x)$ (corresponding to Π_1 and Π_2) are evaluated at a new observation x_0 and $f_1(x_0) = 0{\cdot}32$, $f_2(x_0) = 0{\cdot}56$.

Can the new observation be classified from Π_1 or Π_2 ? *10*

(c) A chemical experiment was performed to investigate the effect of extrusion temperature X_1 and cooling temperature X_2 on the compressibility of a finished product. Knowledge of the process suggested that a model of the form $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + B_{12} X_1 X_2 + \varepsilon$ would satisfactorily explain the variation observed. Two levels of extrusion temperature and two levels of cooling temperature were chosen and all four of the combinations were performed. Each of the four experiments was carried out four times and the data yielded the following information :

ANOVA

S.V	d.f	S.S	MSS
Due to reg.	–	881·2500	
β_0	1	798·0625	
β_1	1	18·0625	
β_2	–	–	
β_{12}	–	5·0625	
Residual	–	–	
Total	16	921·000	

Using $\alpha = 0{\cdot}05$, examine the following questions : *10*

(i) Is the overall regression equation statistically significant ?

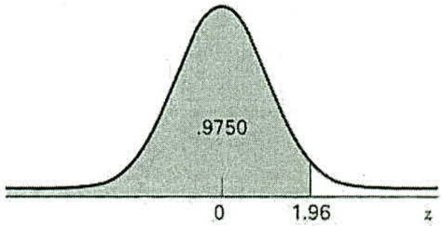
(ii) Are all β significant ?

(Tables 2(a) and 2(b) are provided at the end.)

(d) For a $p \times p$ Latin square with rows (α_i), columns (β_k) and treatments (τ_j) fixed, obtain least squares estimators of α_i, β_k and τ_j, i, j, k = 1, ... p. Derive the missing value formula (when just one observation is missing) for the Latin square design. *10*

TABLE 1(a)

TABLE D Normal Curve Areas $P(z \le z_0)$. Entries in the Body of the Table are Areas Between $-\infty$ and z



z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	0.00	z
−3.80	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	−3.80
−3.70	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	−3.70
−3.60	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002	−3.60
−3.50	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	−3.50
−3.40	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	−3.40
−3.30	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005	−3.30
−3.20	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007	−3.20
−3.10	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010	−3.10
−3.00	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013	−3.00
−2.90	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019	−2.90
−2.80	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026	−2.80
−2.70	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035	−2.70
−2.60	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047	−2.60
−2.50	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062	−2.50
−2.40	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082	−2.40
−2.30	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107	−2.30
−2.20	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139	−2.20
−2.10	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179	−2.10
−2.00	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228	−2.00
−1.90	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287	−1.90
−1.80	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359	−1.80
−1.70	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446	−1.70
−1.60	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548	−1.60
−1.50	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668	−1.50
−1.40	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808	−1.40
−1.30	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968	−1.30
−1.20	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151	−1.20
−1.10	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357	−1.10
−1.00	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587	−1.00
−0.90	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841	−0.90
−0.80	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119	−0.80
−0.70	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420	−0.70
−0.60	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743	−0.60
−0.50	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085	−0.50
−0.40	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446	−0.40
−0.30	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821	−0.30
−0.20	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207	−0.20
−0.10	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602	−0.10
0.00	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000	0.00

TABLE 1(b)

TABLE D (*continued*)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	0.00
0.10	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	0.10
0.20	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	0.20
0.30	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	0.30
0.40	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	0.40
0.50	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	0.50
0.60	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	0.60
0.70	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852	0.70
0.80	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	0.80
0.90	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	0.90
1.00	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	1.00
1.10	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	1.10
1.20	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	1.20
1.30	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	1.30
1.40	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1.40
1.50	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1.50
1.60	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1.60
1.70	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1.70
1.80	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1.80
1.90	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1.90
2.00	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	2.00
2.10	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	2.10
2.20	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	2.20
2.30	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	2.30
2.40	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	2.40
2.50	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	2.50
2.60	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	2.60
2.70	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	2.70
2.80	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	2.80
2.90	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	2.90
3.00	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	3.00
3.10	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	3.10
3.20	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	3.20
3.30	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	3.30
3.40	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	3.40
3.50	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	3.50
3.60	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	3.60
3.70	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	3.70
3.80	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	3.80

TABLE 2(a)

TABLE G (*continued*)

$F_{.95}$

Denominator Degrees of Freedom	Numerator Degrees of Freedom								
	1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

TABLE 2(b)

TABLE G (continued)

Denominator Degrees of Freedom	Numerator Degrees of Freedom									
	10	12	15	20	24	30	40	60	120	∞
1	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00