I.F.S. EXAM-(M)2017

वियोज्य *DETACHABLE*

MATHEMATICS

Paper II

Time Allowed : Three Hours	*Maximum Marks : 200*

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.

Question Nos. **1** and **5** are compulsory. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two **Sections A** and **B**.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question / part is indicated against it.

Answers must be written in **ENGLISH** only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

SECTION 'A'

1. (a) Prove that every group of order four is Abelian. 8

1. (b) A function $f : \mathbb{R} \to \mathbb{R}$ is defined as below :

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 1-x & \text{if } x \text{ is irrational} \end{cases}$$

Prove that f is continuous at $x = \frac{1}{2}$ but discontinuous at all other points in $\mathbb{R}$. 10

1. (c) If $f(z) = u(x, y) + iv(x, y)$ is an analytic function of $z = x + iy$ and $u + 2v = x^3 - 2y^3 + 3xy(2x - y)$ then find $f(z)$ in terms of z. 8

1. (d) Solve by simplex method the following LPP :

Minimize $Z = x_1 - 3x_2 + 2x_3$

subject to the constraints

$3x_1 - x_2 + 2x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 0$

and $x_1, x_2, x_3 \geq 0$ 14

2. (a) Let G be the set of all real numbers except -1 and define $a*b = a + b + ab$ $\forall\ a, b \in G$. Examine if G is an Abelian group under $*$. 10

2. (b) Let H and K are two finite normal subgroups of co-prime order of a group G. Prove that $hk = kh\ \forall h \in H$ and $k \in K$. 10

2. (c) Let A be an ideal of a commutative ring R and $B = \{x \in R : x^n \in A$ for some positive integer $n\}$. Is B an ideal of R ? Justify your answer. 10

2. (d) Prove that the ring $\mathbb{Z}[i] = \{a + ib : a, b \in \mathbb{Z},\ i = \sqrt{-1}\}$ of Gaussian integers is a Euclidean domain. 10

3. (a) Evaluate $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ given that

$$f(x, y) = \begin{cases} x^2 \tan^{-1}\dfrac{y}{x} - y^2 \tan^{-1}\dfrac{x}{y} & \text{if } xy \neq 0 \\ 0 & \text{, otherwise} \end{cases}$$ 10

3. (b) Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the condition $\dfrac{x^2}{4} + \dfrac{y^2}{5} + \dfrac{z^2}{25} = 1$. 10

3. (c) Prove that $\int_0^\infty \frac{\sin x}{x} dx$ is convergent but not absolutely convergent. 12

3. (d) Find the volume of the region common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. 8

4. (a) Prove by the method of contour integration that $\int_0^\pi \frac{1 + 2\cos\theta}{5 + 4\cos\theta} d\theta = 0$. 12

4. (b) Find the sum of residues of $f(z) = \dfrac{\sin z}{\cos z}$ at its poles inside the circle $|z| = 2$. 8

4.(c) Evaluate $\int_{x=0}^{\infty} \int_{y=0}^{x} x\, e^{-x^2/y}\, dy\, dx$ 8

4.(d) A computer centre has four expert programmers. The centre needs four application programs to be developed. The head of the centre after studying carefully the programs to be developed, estimates the computer times in hours required by the experts to the application programs as follows :

Programs

Programmer	A	B	C	D
P_1	5	3	2	8
P_2	7	9	2	6
P_3	6	4	5	7
P_4	5	7	7	8

Assign the programs to the programmers in such a way that total computer time is least. 12

SECTION 'B'

5.(a) Form the partial differential equation by eliminating arbitrary functions φ and ψ from the relation $z = \varphi(x^2 - y) + \psi(x^2 + y)$. 8

5.(b) Write a BASIC program to compute the multiplicative inverse of a non-singular square matrix. 12

5.(c) A uniform rectangular parallelopiped of mass M has edges of lengths $2a$, $2b$, $2c$. Find the moment of inertia of this rectangular parallelopiped about the line through its centre parallel to the edge of length $2a$. 10

5.(d) Evaluate $\int_0^1 e^{-x^2} dx$ using the composite trapezoidal rule with four decimal precision, i.e., with the absolute value of the error not exceeding 5×10^{-5}. 10

6.(a) Solve the partial differential equation :

$$(x-y)\frac{\partial z}{\partial x} + (x+y)\frac{\partial z}{\partial y} = 2xz$$ 8

6.(b) Find the surface which is orthogonal to the family of surfaces $z(x+y) = c(3z+1)$ and which passes through the circle $x^2 + y^2 = 1$, $z = 1$. 8

6.(c) Find complete integral of $xp - yq = xqf(z - px - qy)$ where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. 12

6.(d) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. It is released from rest from this position, find the displacement $y(x, t)$. 12

7. (a) Find the real root of the equation $x^3 + x^2 + 3x + 4 = 0$ correct up to five places of decimal using Newton-Raphson method. 10

7. (b) A river is 80 metre wide, the depth y, in metre, of the river at a distance x from one bank is given by the following table :

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

Find the area of cross-section of the river using Simpson's $\frac{1}{3}$rd rule. 10

7. (c) Find y for $x = 0{\cdot}2$ taking $h = 0{\cdot}1$ by modified Euler's method and compute the error, given that : $\frac{dy}{dx} = x + y,\ y(0) = 1.$ 10

7. (d) Assuming a 32 bit computer representation of signed integers using 2's complement representation, add the two numbers -1 and -1024 and give the answer in 2's complement representation. 10

8. (a) Consider a mass m on the end of a spring of natural length l and spring constant k. Let y be the vertical coordinate of the mass as measured from the top of the spring. Assume that the mass can only move up and down in the vertical direction. Show that

$$L = \tfrac{1}{2}my'^2 - \tfrac{1}{2}k(y-l)^2 + mgy$$

Also determine and solve the corresponding Euler-Lagrange equations of motion. 12

8. (b) Find the streamlines and pathlines of the two dimensional velocity field :

$u = \frac{x}{1+t},\ v = y,\ w = 0.$ 8

8. (c) The velocity vector in the flow field is given by

$\vec{q} = (az - by)\hat{i} + (bx - cz)\hat{j} + (cy - ax)\hat{k}$

where a, b, c are non-zero constants. Determine the equations of vortex lines. 8

8. (d) Solve Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions

$u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\left(\frac{n\pi x}{l}\right).$ 12
